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SUBPIXEL REGISTRATION

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SUBPIXEL REGISTRATION

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ABSTRACT

A method of subpixel image registration is proposed that employs a model of the correlation surface in the vicinity of the registration point.

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1. Introduction

Registration (or matching or correlation) is the process of determining the geometric transformation which is required in order to conform a small image with a large image; see [1]. The simplest transformation is pure translation, where the matching determines the translational offset of the small image in the large one. For each possible offset one defines an object function, whose extremum (maximum or minimum depending on the function) defines the desired matching. For this area (or template) correlation, in contrast to feature-space correlation, the object function, e.g., cross-correlation or sum of absolute differences, directly utilizes the pixel (gray level) values of the two images. The object function is defined only for a discrete grid of offsets between the two images, whereas the true offset of the registration may lie somewhere in between these grid points. If the geometric distortion between the two images is sufficiently small, subpixel accuracy in the registration can be achieved.

In order to achieve subpixel registration some form of interpolation must be used. A straightforward method is to fit some analytic surface to the object function samples in the vicinity of the extremum, and then evaluate analytically the location of the extremum of that surface; see [2, 3]. A similar method, applied by the Canadian Center for Remote Sensing, is to compute the registration point as a weighted sum of neighboring object functions; see [4], where other methods, such as Fourier techniques, are also mentioned.

The problem with these methods is that they do not employ a model of the true surface of the object function in the neighborhood of the registration point.

The actual surface there can be non-analytic (cusped) and asymmetric with respect to the extremum point. In what follows we suggest that better subpixel accuracy in the registration can be achieved if a good model of the correlation surface is employed.

2. The Method

Our main idea, then, is to achieve subpixel accuracy by correlating a small grid of samples of the object function, in the neighborhood of the registration point, with a model of the correlation surface which incorporates its (possible) non-analytic and asymmetric nature. The steps for determining the subpixel registration point are as follows:

(1.) First, the approximate (pixel) registration point, P , is found by the usual method: The object function between the small image and the large image is calculated at the grid points. Then the point P where this function achieves its (global) extremal value is found.

(2.) Next, a model for the correlation surface around P is constructed as follows: A patch from the large image centered at P and of the size of the small image is created. Then the object function between the patch and the large image (the "auto-object function") is calculated on a small grid (say, 5×5 or 7×7) around P . The model surface is passed through these points and is approximated by bilinear or spline interpolation between them. Note that the extremum of the model surface is precisely at P .

(3.) Finally, the subpixel registration point Q is found by correlating a small discrete sample (say 3×3 or 5×5 around P) of the object function calculated in (1), with the continuous model surface defined in (2). The mismatch between the grid and the surface defines the offset Q , the subpixel registration point, and P .

If there exists radiometric (gray level) distortion between the small and large images, then the correlations in (1) and (3) are carried out after normalization with respect to the mean and variance values.

3. An Example

In order to demonstrate the method and its performance, we solve below a simple one-dimensional example. The "exact" picture is assumed to be

$$f(x) = \sin(mx) \quad 0 \leq x \leq \pi \quad (1)$$

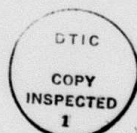
where m is some integer. The large image consists of $N + 1$ pixels centered at

$$x_i = ih = i \frac{\pi}{N}, \quad i = 0, 1, \dots, N \quad (2)$$

with gray levels

$$f_i = \int_{x_i - \frac{h}{2}}^{x_i + \frac{h}{2}} \sin(mx) dx = \frac{1}{m} \left[\cos mh \left(i - \frac{1}{2} \right) - \cos mh \left(i + \frac{1}{2} \right) \right] \quad (3)$$

$$= h \sin(mx_i) \operatorname{sinc} \left(\frac{mh}{2} \right)$$



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The small image is created by sampling the following continuous function:

$$\begin{aligned} g(x) &= \sin(m + \delta m)(x + \delta x) + \epsilon(x) \\ &\cong \sin(mx + m\delta x + x\delta m) + \epsilon(x) \end{aligned} \quad (4)$$

where δm introduces a geometric distortion, and δx introduces a subpixel shift between the two images. $\epsilon(x)$ is the noise. At the registration point, $x = x_{\text{reg}}$, we require that, without noise, $f(x_{\text{reg}}) = g(x_{\text{reg}})$, i.e., $x_{\text{reg}}\delta m + m\delta x = 0$ and therefore

$$g(x) = \sin(m + \delta m)\left(x - \frac{\delta m}{m}x_{\text{reg}}\right) + \epsilon(x) \quad (5)$$

The small image consists of $2j_w + 1$ pixels centered at

$$x_j = (j_0 + \delta j + j)h, \quad j = -j_w, \dots, j_w \quad (6)$$

where

$$x_{\text{reg}} = (j_0 + \delta j)h \quad (7)$$

is the center of the small image. Hence, the gray levels of the pixels of the small image are

$$\begin{aligned} g_j &= \int_{x_j - \frac{h}{2}}^{x_j + \frac{h}{2}} g(x) dx = -\frac{1}{m + \delta m} \cos(m + \delta m)\left(x - \frac{\delta m}{m}x_{\text{reg}}\right) \Big|_{x_j - \frac{h}{2}}^{x_j + \frac{h}{2}} \\ &= \frac{1}{m + \delta m} \left[\cos(m + \delta m)\left(x_j - \frac{h}{2} - \frac{\delta m}{m}x_{\text{reg}}\right) \right. \\ &\quad \left. - \cos(m + \delta m)\left(x_j + \frac{h}{2} - \frac{\delta m}{m}x_{\text{reg}}\right) \right] + \epsilon_j \end{aligned} \quad (8)$$

Once the gray levels of the large (f_i), and small (g_j) images are given, we can calculate the cross-object functions defined as follows:

$$\begin{aligned} h_{fg}^{(1)}(k) &= \frac{1}{n_w} \sum_{j=-j_w}^{j_w} |f_{k+j} - g_j| \\ h_{fg}^{(2)}(k) &= \sum_{j=-j_w}^{j_w} f_{k+j} g_j / \left(\sum_{j=-j_w}^{j_w} f_{k+j}^2 \sum_{j=-j_w}^{j_w} g_j^2 \right)^{1/2} \end{aligned} \quad (9)$$

where $n_w = 2j_w + 1$, $j_w < k < N - j_w$.

Next we create the auto-object function as follows: Assume that the cross-object function defined in Eq. (9) assumes its global extremal value (minimum for $h_{fg}^{(1)}$ and maximum for $h_{fg}^{(2)}$) at the grid point $k = k^*$. Then the auto-object functions are defined as follows:

$$\begin{aligned} h_{ff}^{(1)}(k) &= \frac{1}{n_w} \sum_{j=-j_w}^{j_w} |f_{k+j} - f_{k^*+j}| \\ h_{ff}^{(2)}(k) &= \sum_{j=-j_w}^{j_w} f_{k+j} f_{k^*+j} / \left(\sum_{j=-j_w}^{j_w} f_{k+j}^2 \sum_{j=-j_w}^{j_w} f_{k^*+j}^2 \right)^{1/2} \end{aligned} \quad (10)$$

where n_w and the range of k are defined above, and the limits of summation are as in Eq. (9). Clearly h_{ff} assumes its extremal value precisely at $k = k^*$, where $h_{ff}^{(1)}(k^*) = 0$ and $h_{ff}^{(2)}(k^*) = 1$.

The subpixel registration can now be found by the two methods described in Sections 1 and 2 above:

Method 1

Select a set of grid points (say 3 or 5) around $k = k^*$, where h_{fg} assumes its global extremal value. Fit an analytic function (say, quadratic or quartic)

through these points. Finally, calculate analytically the point $x = \xi_{\text{reg}}$ where the derivative of the analytic function vanishes. The point $x = \xi_{\text{reg}}$ is the sub-pixel registration point, and the fractional error due to the first method is

$$\delta_1 = \frac{|\xi_{\text{reg}} - x_{\text{reg}}|}{h} \quad (11)$$

Method 2

Here the registration point is found by correlating the cross-object function $h_{fg}(k)$, Eq. (9), with the auto-object function $h_{ff}(k)$, Eq. (10), around the grid point $k = k^*$. We first continue the discrete function $h_{ff}(k)$ to a continuous function of s whose origin is located at $k = k^*$, i.e.,

$$s = (k - k^*)h \quad (12)$$

where h is the pixel size defined in Eq. (2). The continuous function $\tilde{h}_{ff}(s)$ coincides with $h_{ff}(k)$ at the grid points, and is interpolated (say, linearly) between these points.

Next we construct a continuous object function $H(s)$ between $\tilde{h}_{ff}(s)$ and $h_{fg}(k)$ defined as follows:

$$\begin{aligned} H^{(1)}(s) &= \frac{1}{n_h} \sum_{j=-j_h}^{j_h} |h_{fg}^{(1)}(k^* + j) - h^{(1)}(s + jh)| \\ H^{(2)}(s) &= \sum_{j=-j_h}^{j_h} h_{fg}^{(2)}(k^* + j) \tilde{h}^{(2)}(s + jh) / D \\ D &= \left\{ \sum_{j=-j_h}^{j_h} \{h_{fg}^{(2)}(k^* + j)\}^2 \sum_{j=-j_h}^{j_h} \{\tilde{h}^{(2)}(s + jh)\}^2 \right\}^{1/2} \end{aligned} \quad (13)$$

where $j_h = 1$ or 2 , and $n_j = 2j_h + 1$.

The registration point is at $s = s^*$ where $H(s)$ assumes its extremal value (minimum for $H^{(1)}(s^*)$ and maximum for $H^{(2)}(s^*)$). Since $H(s)$ is not given in closed form, s^* cannot be found analytically, but has to be evaluated numerically (say, by Newton's method). In the coordinate system of the large image, the sub-pixel registration point is at $x = \eta_{\text{reg}}$, where

$$\eta_{\text{reg}} = k^* h - s^* \quad (14)$$

and the fractional error due to the second method is

$$\delta_2 = \frac{|\eta_{\text{reg}} - x_{\text{reg}}|}{h} \quad (15)$$

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